

Formal Semantics for MCJ

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1 Preliminaries

1.1 Metavariables

- Expressions or mappings: e, d, r
- Field names: f, g
- Variables: x
- Method names: m
- Values: v
- Method definitions: M, N
- Type names: C, D
- Types: I, J, K, U, V, W
- Non-typeof types: O, P, Q, R, S
- Types that are Object or a type application: A, B
- Ground types (contain no type variables): G
- Type variables: X, Y, Z
- Mappings (field name \mapsto value): Φ

1.2 Syntax

CL	::=	class C($X \triangleleft (T, T)$) : A \triangleleft A { $\bar{T} \bar{f} \bar{e}; \bar{M}; \bar{T} \bar{f}; \bar{M}$ }	class declaration
M	::=	T m($\bar{T} \bar{x}$) {e}	method definition
e	::=	x	variable reference
		e.f	field access
		e.m(\bar{e})	method invocation
		new _C (\bar{r}) (\bar{e})	instance creation
		(T)e	cast
		R	type reference
T	::=	R	non-typeof type
		typeof [R]	typeof application
R	::=	X	type variable
		A	
A	::=	C(\bar{R})	type application
		Object	

2 Evaluation

2.1 Congruence Rules

Rules for congruence

$$\begin{array}{c}
\frac{e \rightarrow e'}{e.f \rightarrow e'.f} \text{ [C-FIELD]} \quad \frac{e \rightarrow e'}{e.m(\bar{d}) \rightarrow e'.m(\bar{d})} \text{ [C-RCVR]} \quad \frac{e \rightarrow e'}{\text{new}_{\mathbb{C}}(\bar{S})(\bar{e}) \rightarrow \text{new}_{\mathbb{C}}(\bar{S})(\bar{e}')} \text{ [C-NEW]} \\
\frac{\bar{e} \rightarrow \bar{e}'}{d.m(\bar{e}) \rightarrow d.m(\bar{e}')} \text{ [C-ARG]} \quad \frac{\bar{e} \rightarrow \bar{e}'}{\{\bar{f} \mapsto \bar{e}\}_{\mathbb{G}} \rightarrow \{\bar{f} \mapsto \bar{e}'\}_{\mathbb{G}}} \text{ [C-MAP]} \quad \frac{e \rightarrow e'}{(\mathbb{T})e \rightarrow (\mathbb{T})e'} \text{ [C-CAST]}
\end{array}$$

$\bar{e} \rightarrow \bar{e}'$ means one of the e 's step.

2.2 Reduction Rules

$$\text{Object} \rightarrow \{\} \text{Object} \text{ [R-OBJECT]} \quad \Phi_{\mathbb{G}}.f \rightarrow \Phi_{\mathbb{G}}(f) \text{ [R-FIELD]}$$

$$\frac{\text{fields}(\mathbb{C}(\bar{S})) = \bar{\mathbb{T}} \bar{f}}{\text{new}_{\mathbb{C}}(\bar{S})(\bar{e}) \rightarrow \{\bar{f} \mapsto \bar{e}\}_{\mathbb{C}(\bar{S})}} \text{ [R-NEW]} \quad \frac{\text{field-vals}(\text{typeOf}[\mathbb{C}(\bar{S})]) = \bar{\mathbb{T}} \bar{f} \bar{e}}{\mathbb{C}(\bar{S}) \rightarrow \{\bar{f} \mapsto \bar{e}\}_{\text{typeOf}[\mathbb{C}(\bar{S})]}} \text{ [R-CLASS]}$$

$$\frac{\emptyset \vdash \mathbb{G} <: \mathbb{T}}{(\mathbb{T})\Phi_{\mathbb{G}} \rightarrow \Phi_{\mathbb{G}}} \text{ [R-CAST]} \quad \frac{\text{mbody}(\mathbb{m}, \mathbb{G}) = (\bar{x}, e_0)}{\Phi_{\mathbb{G}}.m(\bar{d}) \rightarrow [\bar{x} \mapsto \bar{d}, \text{this} \mapsto \Phi_{\mathbb{G}}]e_0} \text{ [R-INVK]}$$

3 Typing

3.1 Subtyping

Subtyping rules

$$\Delta \vdash T <: T \text{ [S-REFLEX]}$$

$$\frac{(X \triangleleft T) \in \Delta \text{ [S-VAR]}}{\Delta \vdash X <: T} \quad \frac{\Delta \vdash V <: T \quad \Delta \vdash T <: U \text{ [S-TRANS]}}{\Delta \vdash V <: U}$$

$$\frac{CT(I) = \text{interface } I(X \triangleleft (L, L')) : K \sqsubseteq \bar{J} \{ \dots \} \text{ [S-INTER]}}{\Delta \vdash I(\bar{S}) <: [\bar{X} \mapsto \bar{S}]J} \quad \frac{CT(C) = \text{class } C(X \triangleleft (I, J)) : B \triangleleft A \text{ [S-KIND]}}{\Delta \vdash \text{typeOf}[C(\bar{S})] <: [\bar{X} \mapsto \bar{S}]B}$$

$$\frac{CT(C) = \text{class } C(X \triangleleft (I, J)) : B \triangleleft A \text{ [S-IMPL]}}{\Delta \vdash C(\bar{S}) <: [\bar{X} \mapsto \bar{S}]I} \quad \frac{CT(C) = \text{class } C(X \triangleleft (I, J)) : B \triangleleft A \text{ [S-SUPER]}}{\Delta \vdash C(\bar{S}) <: [\bar{X} \mapsto \bar{S}]A}$$

$$\frac{CT(C) = \text{class } C(X \triangleleft (I, J)) : B \triangleleft A \text{ [S-CIMPL]}}{\Delta \vdash \text{typeOf}[C(\bar{S})] <: [\bar{X} \mapsto \bar{S}]I'}$$

3.2 Type Bounds

$$\text{bound}_\Delta(X) = \Delta(X) \quad \text{bound}_\Delta(N) = N \quad \text{bound}_\Delta(\text{typeOf}[X]) = \Delta(\text{typeOf}[X])$$

3.3 Well-Formed Constructs

Well-formedness checks

$$\Delta \vdash \text{Object ok [WF-OBJECT]} \quad \frac{\Delta \vdash T \text{ ok}}{\Delta \vdash \text{typeOf}[T] \text{ ok}} \text{ [WF-TYPEOF]} \quad \frac{\Delta \vdash X <: T}{\Delta \vdash X \text{ ok}} \text{ [WF-VAR]}$$

$$\frac{\Delta \vdash \bar{S} <: [\bar{X} \mapsto \bar{S}]\bar{I} \quad \Delta \vdash \text{typeOf}[\bar{S}] <: [\bar{X} \mapsto \bar{S}]\bar{J} \quad \Delta \vdash \bar{S} \text{ ok}}{\Delta \vdash C(\bar{S}) \text{ ok}} \text{ [WF-CLASS]}$$

$$\frac{\begin{array}{l} \text{params}(G) = \bar{X} \sqsubseteq (\bar{I}, \bar{J}) \\ \Delta = \bar{X} \triangleleft \bar{I}, \text{typeOf}[\bar{X}] \triangleleft \bar{J} \quad \Gamma = \bar{x} : \bar{T}, \text{this} : G \\ \Delta \vdash \bar{T} \text{ ok} \quad \Delta \vdash U \text{ ok} \quad \text{override}(m, \text{super}(G), \bar{T} \rightarrow U) \\ \Delta; \Gamma \vdash e : W \quad \Delta \vdash W <: U \end{array}}{U \text{ m}(\bar{T} \bar{x})\{e\} \text{ ok in } G} \text{ [WF-METHOD]}$$

$$\frac{\begin{array}{l} \bar{M} \text{ ok in } \text{typeOf}[C(\bar{X})] \quad \bar{N} \text{ ok in } C(\bar{X}) \quad \Delta = \bar{X} \triangleleft \bar{I}, \text{typeOf}[\bar{X}] \triangleleft \bar{J} \\ \Delta \vdash B \text{ ok} \quad \Delta \vdash A \text{ ok} \quad \Delta \vdash I \text{ ok} \quad \Delta \vdash J \text{ ok} \quad \Delta \vdash \bar{T} \text{ ok} \quad \Delta \vdash \bar{W} \text{ ok} \\ \text{fields}(B) \subseteq \bar{T} \bar{f} \quad \Delta; \emptyset \vdash \bar{e} : \bar{T}' \quad \emptyset \vdash \bar{T}' <: \bar{T} \\ \forall \text{new}_D(\bar{S})(\bar{d}) \in \bar{e}, \bar{M}, \bar{N}. D = C \end{array}}{\text{class } C(X \triangleleft (I, J)) : B \triangleleft A \{ \bar{T} \bar{f} \bar{e}; \bar{M}; \bar{W} \bar{g}; \bar{N} \} \text{ ok}} \text{ [WF-CLASSDEF]}$$

3.4 Expression Typing

Rules for typing expressions.

$$\begin{array}{c}
\Delta; \Gamma \vdash \mathbf{x} : \Gamma(\mathbf{x}) \text{ [T-VAR]} \quad \frac{\Delta \vdash \mathbf{T} \text{ ok}}{\Delta; \Gamma \vdash \mathbf{e} : \mathbf{U}} \text{ [T-CAST]} \quad \frac{\Delta \vdash \mathbf{S} \text{ ok}}{\Delta; \Gamma \vdash \mathbf{S} : \text{typeOf}[\mathbf{S}]} \text{ [T-CLASS]} \\
\\
\frac{\Delta; \Gamma \vdash \mathbf{e} : \mathbf{U} \quad \text{fields}(\text{bound}_\Delta(\mathbf{U})) = \bar{\mathbf{T}} \bar{\mathbf{f}}}{\Delta; \Gamma \vdash \mathbf{e} . \mathbf{f}_i : \mathbf{T}_i} \text{ [T-FIELD]} \quad \frac{\text{fields}(\mathbf{G}) = \bar{\mathbf{T}} \bar{\mathbf{f}} \quad \Delta; \Gamma \vdash \bar{\mathbf{e}} : \bar{\mathbf{S}} \quad \Delta \vdash \bar{\mathbf{S}} <: \bar{\mathbf{T}}}{\Delta; \Gamma \vdash \{\bar{\mathbf{f}} \mapsto \bar{\mathbf{e}}\}_{\mathbf{G}} : \mathbf{G}} \text{ [T-MAPPING]} \\
\\
\frac{\Delta; \Gamma \vdash \mathbf{r} : \mathbf{W} \quad \Delta; \Gamma \vdash \bar{\mathbf{e}} : \bar{\mathbf{V}} \quad \Delta \vdash \bar{\mathbf{V}} <: \bar{\mathbf{U}} \quad \text{mtype}(\mathbf{m}, \text{bound}_\Delta(\mathbf{W})) = \bar{\mathbf{U}} \rightarrow \bar{\mathbf{T}}}{\Delta; \Gamma \vdash \mathbf{r} . \mathbf{m}(\bar{\mathbf{e}}) : \mathbf{T}} \text{ [T-INVK]} \\
\\
\frac{\Delta \vdash \mathbf{C}(\bar{\mathbf{S}}) \text{ ok} \quad \text{fields}(\mathbf{C}(\bar{\mathbf{S}})) = \bar{\mathbf{T}} \bar{\mathbf{f}} \quad \Delta \vdash \bar{\mathbf{U}} <: \bar{\mathbf{T}} \quad \Delta; \Gamma \vdash \bar{\mathbf{e}} : \bar{\mathbf{U}}}{\Delta; \Gamma \vdash \text{new}_{\mathbf{C}}(\bar{\mathbf{S}})(\bar{\mathbf{e}}) : \mathbf{C}(\bar{\mathbf{S}})} \text{ [T-NEW]}
\end{array}$$

4 Auxilliary Functions

4.1 Field Lookup

$$\text{fields}(\text{Object}) = \emptyset \text{ [F-OBJECT]} \quad \text{field-vals}(\text{Object}) = \emptyset \text{ [FV-OBJECT]} \quad \text{field-vals}(\mathbf{C}(\bar{\mathbf{S}})) = \emptyset \text{ [FV-CLASS]}$$

$$\frac{CT(\mathbf{C}) = \text{class } \mathbf{C}(X \triangleleft (\mathbf{I}, \mathbf{J})) : \mathbf{B} \triangleleft \mathbf{A} \{ \bar{\mathbf{T}} \bar{\mathbf{f}} \bar{\mathbf{e}}; \bar{\mathbf{M}}; \bar{\mathbf{W}} \bar{\mathbf{g}}; \bar{\mathbf{N}} \} \quad \text{fields}([\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}]\mathbf{A}) = (\bar{\mathbf{U}}\mathbf{h})}{\text{fields}(\mathbf{C}(\bar{\mathbf{R}})) = (\bar{\mathbf{U}}\mathbf{h} \cup [\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}]\bar{\mathbf{W}}\bar{\mathbf{g}})} \text{ [F-CLASS]}$$

$$\frac{CT(\mathbf{C}) = \text{class } \mathbf{C}(X \triangleleft (\mathbf{I}, \mathbf{J})) : \mathbf{B} \triangleleft \mathbf{A} \{ \bar{\mathbf{T}} \bar{\mathbf{f}} \bar{\mathbf{e}}; \bar{\mathbf{M}}; \bar{\mathbf{W}} \bar{\mathbf{g}}; \bar{\mathbf{N}} \}}{\text{fields}(\text{typeOf}[\mathbf{C}(\bar{\mathbf{R}})]) = [\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}]\bar{\mathbf{T}} \bar{\mathbf{f}}} \text{ [F-TYPEOF]}$$

$$\frac{CT(\mathbf{C}) = \text{class } \mathbf{C}(X \triangleleft (\mathbf{I}, \mathbf{J})) : \mathbf{B} \triangleleft \mathbf{A} \{ \bar{\mathbf{T}} \bar{\mathbf{f}} \bar{\mathbf{e}}; \bar{\mathbf{M}}; \bar{\mathbf{W}} \bar{\mathbf{g}}; \bar{\mathbf{N}} \}}{\text{field-vals}(\text{typeOf}[\mathbf{C}(\bar{\mathbf{R}})]) = [\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}]\bar{\mathbf{T}} \bar{\mathbf{f}} \bar{\mathbf{e}}} \text{ [FV-TYPEOF]}$$

4.2 Method Lookup

$$\frac{\mathbf{U} \mathbf{m}(\bar{\mathbf{T}} \bar{\mathbf{x}}) \{ \mathbf{e} \} \in \text{methods}(\mathbf{G})}{\text{mtype}(\mathbf{m}, \mathbf{G}) = \bar{\mathbf{T}} \rightarrow \bar{\mathbf{U}}} \text{ [M-TYPE]} \quad \frac{\mathbf{U} \mathbf{m}(\bar{\mathbf{T}} \bar{\mathbf{x}}) \{ \mathbf{e} \} \in \text{methods}(\mathbf{G})}{\text{mbody}(\mathbf{m}, \mathbf{G}) = (\bar{\mathbf{x}}, \bar{\mathbf{e}})} \text{ [M-BODY]}$$

$$\text{methods}(\text{Object}) = \emptyset \text{ [METHODS-OBJECT]}$$

$$\frac{CT(\mathbf{C}) = \text{class } \mathbf{C}(X \triangleleft (\mathbf{I}, \mathbf{J})) : \mathbf{B} \triangleleft \mathbf{A} \{ \bar{\mathbf{T}} \bar{\mathbf{f}} \bar{\mathbf{e}}; \bar{\mathbf{M}}; \bar{\mathbf{W}} \bar{\mathbf{g}}; \bar{\mathbf{N}} \}}{\text{methods}(\mathbf{C}(\bar{\mathbf{R}})) = ([\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}]\bar{\mathbf{N}} \cup \text{methods}([\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}]\mathbf{A}))} \text{ [METHODS-CLASS]}$$

$$\frac{CT(\mathbf{C}) = \text{class } \mathbf{C}(X \triangleleft (\mathbf{I}, \mathbf{J})) : \mathbf{B} \triangleleft \mathbf{A} \{ \bar{\mathbf{T}} \bar{\mathbf{f}} \bar{\mathbf{e}}; \bar{\mathbf{M}}; \bar{\mathbf{W}} \bar{\mathbf{g}}; \bar{\mathbf{N}} \}}{\text{methods}(\text{typeOf}[\mathbf{C}(\bar{\mathbf{R}})]) = ([\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}]\bar{\mathbf{M}} \cup \text{methods}([\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}]\mathbf{B}))} \text{ [METHODS-TYPEOF]}$$

4.3 Overriding

$$\frac{(\text{mtype}(\mathbf{m}, \mathbf{G}) = \bar{\mathbf{U}} \rightarrow \bar{\mathbf{V}} \text{ implies } \bar{\mathbf{U}} = \bar{\mathbf{W}} \text{ and } \bar{\mathbf{T}} = \bar{\mathbf{V}})}{\text{override}(\mathbf{m}, \mathbf{G}, \bar{\mathbf{W}} \rightarrow \bar{\mathbf{T}})} \text{ [OVERRIDE]}$$

4.4 Paramaters

$$\frac{CT(\mathbf{C}) = \text{class } C(X \triangleleft (I, J)) : B \triangleleft A}{\text{params}(C(\overline{S})) = \overline{X} \sqsubset (\overline{I}, \overline{J})} [\text{PARAMS}] \quad \frac{CT(\mathbf{C}) = \text{class } C(X \triangleleft (I, J)) : B \triangleleft A}{\text{params}(\text{typeof } [C(\overline{S})]) = \overline{X} \sqsubset (\overline{I}, \overline{J})} [\text{PARAMSTYPEOF}]$$

4.5 Super

$$\frac{CT(\mathbf{C}) = \text{class } C(X \triangleleft (I, J)) : B \triangleleft A}{\text{super}(C(\overline{S})) = A} [\text{SUPER}] \quad \frac{CT(\mathbf{C}) = \text{class } C(X \triangleleft (I, J)) : B \triangleleft A}{\text{super}(\text{typeof } [C(\overline{S})]) = B} [\text{SUPERTYPEOF}]$$

5 Soundness Proof

5.1 Subject Reduction

Theorem 1 (Subject Reduction) *If $\emptyset; \emptyset \vdash e : S$ and $e \rightarrow e'$ then $\emptyset; \emptyset \vdash e' : T$ where $\emptyset \vdash T <: S$.*

Proof We prove this by structural induction on the derivation of $e \rightarrow e'$

Case [R-OBJECT]: Immediate.

Case [R-NEW]: Immediate from the premises of [T-NEW] and [R-NEW].

Case [R-CLASS]: Immediate from the premises of [WF-CLASSDEF] and [R-CLASS].

Case [R-CAST]: By [T-CAST], $e = (T)\Phi_{\mathbf{G}}$ must have type T . By [T-MAPPING], $e' = \Phi_{\mathbf{G}}$ must have type G . By hypothesis of the reduction rule, $\emptyset \vdash G <: T$.

Case [R-FIELD]: We know that $e = \{\bar{f} \mapsto \bar{d}\}_{\mathbf{G}}.f_i$ and that $e' = d_i$. Since e must have been typed by [T-FIELD], we know that $fields(\mathbf{G}) = \bar{T} \bar{f}$ and $\emptyset; \emptyset \vdash e : T_i$. Further, since $\{\bar{f} \mapsto \bar{d}\}_{\mathbf{G}}$ must have been typed by [T-MAPPING], we know that $\emptyset; \emptyset \vdash d_i : S$ where $\emptyset \vdash S <: T$.

Case [R-INVK]: We know that $e = \Phi_{\mathbf{G}}.m(\bar{d})$ and that $e' = [\bar{x} \mapsto \bar{d}, \mathbf{this} \mapsto \Phi_{\mathbf{G}}] e_0$. Further, e was typed by [T-INVK] to have type U where $mtype(m, \mathbf{G}) = \bar{T} \rightarrow U$ and $\emptyset; \emptyset \vdash \bar{d} : \bar{T}'$ and $\emptyset \vdash \bar{T}' <: \bar{T}$. As a premise of [R-INVK], we know that $mbody(m, \mathbf{G}) = (\bar{x}, e_0')$. By the lemma (it 'mtype) and (it 'mbody) agree, $\emptyset; \bar{x} : \bar{T}\mathbf{this} : G \vdash e_0' : U'$ where $\emptyset \vdash U' <: U$. Then by the lemma Substitution Preserves Typing, $\emptyset; \emptyset \vdash e' = [\bar{x} \mapsto \bar{d}, \mathbf{this} \mapsto \Phi_{\mathbf{G}}] e_0 : U''$ where $\emptyset \vdash U'' <: U'$.

Case [C-CAST]: Trivial, since $\emptyset; \emptyset \vdash (T)e : T$ for any e

Case [C-MAP]: Immediate from the induction hypothesis and the transitivity of subtyping.

Case [C-NEW]: Immediate from the induction hypothesis and the transitivity of subtyping.

Case [C-ARG]: Immediate from the induction hypothesis and the transitivity of subtyping.

Case [C-RCVR]: We know that $e = e_0.m(\bar{d})$ and $e' = e_0'.m(\bar{d})$. Further, e must have been typed by [T-INVK], which means that $\emptyset; \emptyset \vdash e_0 : W$ for some ground type W , and that $\emptyset; \emptyset \vdash \bar{d} : \bar{V}$ and $\emptyset \vdash \bar{V} <: \bar{U}$, and also $mtype(m, W) = \bar{U} \rightarrow T$. By the induction hypothesis, $\emptyset; \emptyset \vdash e_0' : W'$ where $\emptyset \vdash W' <: W$. Therefore, by lemma Subtyping Preserves Method Typing, $mtype(m, W') = \bar{U} \rightarrow T$ and thus $\emptyset; \emptyset \vdash e_0'.m(\bar{d}) : T$ by [T-INVK].

Case [C-FIELD]: If $e.f_i \rightarrow e'.f_i$ then $e \rightarrow e'$. Further, $e.f_i$ must have been typed by [T-FIELD] to have type T_i . Therefore, by the induction hypothesis, $\emptyset; \emptyset \vdash e : S$ and $\emptyset; \emptyset \vdash e' : S'$ where $\emptyset \vdash S <: S'$. Then, by the lemma Fields are Preserved by Subtypes, $fields(S') = (fields(S) @ \bar{F})$, and by [T-FIELD], $\emptyset; \emptyset \vdash e'.f_i : T_i$.

5.2 Progress

Theorem 2 (Progress) *If $\emptyset; \emptyset \vdash e : S$ then one of the following holds:*

- $e = \{\bar{f} \mapsto \bar{v}\}_{\mathbf{G}}$
- $e \rightarrow e'$
- $e = (S)e'$ and $\emptyset; \emptyset \vdash e' : T$ and $\emptyset \vdash T <: S$.

Proof We prove this by induction over the derivation of $\emptyset; \emptyset \vdash e : S$.

Case [T-VAR]: This is a contradiction, since e is ground

Case [T-CLASS]: In this case $e = C(\bar{S})'$ where $C(\bar{S})'$ is ground. Then lemma Agreement of *field-vals* and *fields params* $(C(\bar{S})') = \bar{X} \sqsubset (\bar{I}, \bar{J})$ for some ????. Further, $field-vals(\mathit{typeOf}[C(\bar{S})']) = \bar{T} \ \bar{f} \ \bar{e}$ for some \bar{T}, \bar{f} and \bar{e} . Therefore, [R-CLASS] applies and $S \rightarrow \{\bar{f} \mapsto [\bar{X} \mapsto \bar{S}']C(\bar{S})'\}_{\mathit{typeOf}[C(\bar{S})']}$.

Case [T-MAPPING]: Either e is already a value, or $e = \{\bar{f} \mapsto \bar{e}\}_{\mathbf{G}}$ where not all of the \bar{e} are values. Then by the induction hypothesis, there is some i such that either $e_i \rightarrow e'_i$, in which case [C-MAPPING] applies, or e_i contains a bad cast, and the case is complete.

Case [T-CAST]: Here there are three cases:

- $e = (S)e'$ where e' is not a mapping. Then [C-CAST] applies.
- $e = (S)e'$ where $\emptyset \vdash T <: S$. Then [R-CAST] applies.
- $e = (S)e'$ where $\emptyset \vdash T <: S$. Then e is a bad cast.

Case [T-NEW]: From the antecedent of [T-NEW] the premise of [R-NEW] applies.

Case [T-INVK]: Here there are two cases:

- $e = r.m(\bar{d})$ where r is not a mapping. Then by the induction hypothesis, either r contains a bad cast or $r \rightarrow r'$ and [C-RCVR] applies
- $e = \Phi_{\mathbf{T}}.m(\bar{d})$ We know from the antecedent that $mtype(m, bound_{\Delta}(T)) = \bar{U} \rightarrow V$ and therefore $mtype(m, T) = \bar{U} \rightarrow V$ since T is ground. Therefore, since *mbody* is defined everywhere *mtype* is defined, $mbody(m, T) = (\bar{x}, e_0)$ for some \bar{x} and e_0 . Thus [R-INVK] applies.

Case [T-FIELD]: Here there are two cases, either the receiver is a mapping or not. In the first, by the antecedent of the typing rule, we can lookup the field successfully and apply [R-FIELD]. Otherwise, we can apply [C-FIELD].

5.3 Type Soundness

Theorem 3 (Type Soundness) *If $\emptyset; \emptyset \vdash e : S$ then one of the following holds:*

- $e \rightarrow^* \{\bar{f} \mapsto \bar{v}\}_{\mathbf{G}}$ where $\emptyset \vdash \mathbf{G} <: S$
- $e \rightarrow^* e'$ where e' is an invalid cast
- e reduces infinitely.

Proof Immediate from Subject Reduction and Progress

5.4 Lemmas

Lemma 1 (Type Substitution Preserves Subtyping) *If $\Delta = \bar{X} \triangleleft \bar{S}$ and $\Delta \vdash U <: T$ and $\emptyset \vdash \bar{S}' <: [\bar{X} \mapsto \bar{S}']\bar{S}$ then $\emptyset \vdash [\bar{X} \mapsto \bar{S}']U <: [\bar{X} \mapsto \bar{S}']T$.*

Proof By induction over the derivation of $\Delta \vdash U <: T$.

Case [S-Reflex]: Immediate.

Case [S-Trans]: Let V be the intermediate type. By the induction hypothesis, $\emptyset \vdash [\bar{X} \mapsto \bar{S}']U <: [\bar{X} \mapsto \bar{S}']V$ and $\emptyset \vdash [\bar{X} \mapsto \bar{S}']V <: [\bar{X} \mapsto \bar{S}']T$ and application of [S-TRANS] completes the case

Case [S-Super]: In this case $U = C(\bar{R})$ and $CT(C) = \text{class } C(\bar{Y} \triangleleft (\bar{I}, \bar{J})) : B \triangleleft A \{ \dots \}$ and $[\bar{Y} \mapsto \bar{R}]A = T$. We must show that $\emptyset \vdash [\bar{X} \mapsto \bar{S}']C(\bar{R}) <: [\bar{X} \mapsto \bar{S}'][\bar{Y} \mapsto \bar{R}]A$. But note that:

$$[\bar{X} \mapsto \bar{S}']C(\bar{R}) = C([\bar{X} \mapsto \bar{S}']\bar{R}) = C([\bar{X} \mapsto \bar{S}'][\bar{Y} \mapsto \bar{R}]\bar{Y}) = C([\bar{Y} \mapsto [\bar{X} \mapsto \bar{S}']\bar{R}]\bar{Y}).$$

Also, $[\bar{X} \mapsto \bar{S}'][\bar{Y} \mapsto \bar{R}]A = [\bar{Y} \mapsto [\bar{X} \mapsto \bar{S}']\bar{R}]A$. So, $\emptyset \vdash C([\bar{X} \mapsto \bar{S}']\bar{R}) <: [\bar{Y} \mapsto [\bar{X} \mapsto \bar{S}']\bar{R}]A$ by [S-SUPER], completing the case.

Case [S-Kind]: Analogous to [S-SUPER].

Case [S-Bound]: In this case, $U = X_i$ and $T = S_i$. Then $[\bar{X} \mapsto \bar{S}']U = S'_i$ and $[\bar{X} \mapsto \bar{S}']T = [\bar{X} \mapsto \bar{S}']S_i$. We are given $\emptyset \vdash S'_i <: [\bar{X} \mapsto \bar{S}']S_i$, completing the case.

Lemma 2 (Type Substitution Preserves Typing) *If $\Delta = \bar{X} \triangleleft \bar{S}$ and $\Delta; \Gamma \vdash e : T$ and $\emptyset \vdash \bar{S}' <: [\bar{X} \mapsto \bar{S}']\bar{S}$ then $\emptyset; [\bar{X} \mapsto \bar{S}']\Gamma \vdash [\bar{X} \mapsto \bar{S}']e : [\bar{X} \mapsto \bar{S}']T$.*

Proof We prove this by induction over the derivation of $\Delta; \Gamma \vdash e : T$.

Case [T-VAR]: $e = v$, $T = \Gamma(v)$ Then $[\bar{X} \mapsto \bar{S}']e = v$ and $[\bar{X} \mapsto \bar{S}']\Gamma(v) = [\bar{X} \mapsto \bar{S}']\Gamma(v)$

Case [T-CLASS]: $e = R$, $T = \text{typeOf } [R]$ If $R \neq X_i$ then substitution has no effect. Otherwise, $[\bar{X} \mapsto \bar{S}']R = S'_i$ and $[\bar{X} \mapsto \bar{S}']\text{typeOf } [R] = \text{typeOf } [S'_i]$

Case [T-CAST]: $e = (R)e'$, $T = R$

Case $R \neq X_i$: Then $[\bar{X} \mapsto \bar{S}']e = (R)[\bar{X} \mapsto \bar{S}']e'$, which still has type R

Case $R = X_i$: Then $[\bar{X} \mapsto \bar{S}']e = (S_i)[\bar{X} \mapsto \bar{S}']e'$, which has type $S_i = [\bar{X} \mapsto \bar{S}']R$.

Case [T-MAPPING]: $e = \{f \mapsto e'\}_{G}$, $T = G$ Then $[\bar{X} \mapsto \bar{S}']e = \{f \mapsto [\bar{X} \mapsto \bar{S}']e'\}_{G}$ since G is ground and unaffected by the substitution. Then by [T-MAPPING], $\emptyset; [\bar{X} \mapsto \bar{S}']\Gamma \vdash [\bar{X} \mapsto \bar{S}']e : [\bar{X} \mapsto \bar{S}']G$

Case [T-NEW]: $e = \text{new}_C(\bar{R})(\bar{e}')$, $T = C(\bar{R})$ Then $[\bar{X} \mapsto \bar{S}']e = \text{new}_C([\bar{X} \mapsto \bar{S}']\bar{R})([\bar{X} \mapsto \bar{S}']\bar{e}')$. By the induction hypothesis, $\emptyset; [\bar{X} \mapsto \bar{S}']\Gamma \vdash [\bar{X} \mapsto \bar{S}']\bar{e}' : [\bar{X} \mapsto \bar{S}']\bar{U}$ where $\emptyset; \Gamma \vdash \bar{e}' : \bar{U}$. By Type Subst. Preserves Subtyping, $\emptyset \vdash [\bar{X} \mapsto \bar{S}']\bar{U} <: [\bar{X} \mapsto \bar{S}']\bar{V}$ where $\bar{V} \ \bar{f} = \text{fields}(C(\bar{R}))$

Case [T-FIELD]: $e = e' . f_i$, $T = T_i$ where $\text{fields}(\text{bound}_\Delta(U)) = \bar{T} \ \bar{f}$ and $\Delta; \Gamma \vdash e' : U$. By the induction hypothesis, $\emptyset; [\bar{X} \mapsto \bar{S}']\Gamma \vdash [\bar{X} \mapsto \bar{S}']e' : [\bar{X} \mapsto \bar{S}']U$ There are two subcases:

Case $U = X_i$: Then $[\bar{X} \mapsto \bar{S}']\text{fields}(\text{bound}_\Delta(U)) = [\bar{X} \mapsto \bar{S}']S_i \ \bar{f}$; $S'_i \ \bar{f}$ by assumption and $S'_i = \text{bound}_\emptyset([\bar{X} \mapsto \bar{S}']U)$, and because field shadowing is disallowed $[\bar{X} \mapsto \bar{S}']\text{fields}(\text{bound}_\Delta(U)) \subseteq \text{fields}(\text{bound}_\emptyset([\bar{X} \mapsto \bar{S}']U))$. Then by [T-FIELD], $\emptyset; [\bar{X} \mapsto \bar{S}']\Gamma \vdash [\bar{X} \mapsto \bar{S}']e : [\bar{X} \mapsto \bar{S}']T$

Case $U \neq X_i$ for any i : Then $\text{bound}_\Delta(U) = U$. Then by definition of fields , $[\bar{X} \mapsto \bar{S}']\text{fields}(U) = \text{fields}([\bar{X} \mapsto \bar{S}']U)$. Then by [T-FIELD] $\emptyset; [\bar{X} \mapsto \bar{S}']\Gamma \vdash [\bar{X} \mapsto \bar{S}']e : [\bar{X} \mapsto \bar{S}']T$

Case [T-INVK]: This case is almost identical to the previous case, except that although method overriding is allowed, type invariance of methods is required by [VALIDOVERRIDE].

Lemma 3 (Substitution Preserves Typing) *If $\Delta; \Gamma \vdash e : T$ and $\Gamma = \bar{x} : \bar{S}$ and $\Delta; \emptyset \vdash \bar{d} : \bar{U}$ and $\Delta \vdash \bar{U} <: \bar{S}$ then $\Delta; \emptyset \vdash [\bar{x} \mapsto \bar{d}]e : T$ where $\Delta \vdash T' <: T$.*

Proof We prove this by induction over the derivation of $\Delta; \Gamma \vdash e : T$.

Case [T-VAR]: $e = x_i$, $T = S_i$ But $\Delta; \emptyset \vdash d_i : U_i$ and $\Delta \vdash U_i <: S_i$, completing the case.

Case [T-CLASS]: Immediate since the expression casted does not impact the type.

Case [T-CAST]: Immediate since $[\bar{x} \mapsto \bar{d}] e = e$.

Case [T-MAPPING]: $e = \{\bar{f} \mapsto \bar{e}'\}_{\mathbf{G}}$, $T = \mathbf{G}$ By the induction hypothesis, all the substituted e' will have subtypes of their original types, so [T-MAPPING] applies.

Case [T-NEW]: $e = \text{new}_{\mathbf{C}}(\bar{\mathbf{R}})(\bar{e}')$, $T = \mathbf{C}(\bar{\mathbf{R}})$ Immediate by the application of the induction hypothesis to \bar{e}' .

Case [T-FIELD]: $e = e'.f_i$, $T = T_i$ Let $\Delta; \Gamma \vdash e' : R$. By the induction hypothesis, $\Delta; \emptyset \vdash [\bar{x} \mapsto \bar{d}]e' : R'$ where $\Delta \vdash R' <: R$. Then, by the definition of *fields*, $fields(R) \subseteq fields(R')$. Therefore, [T-FIELD] applies and $\Delta; \emptyset \vdash [\bar{x} \mapsto \bar{d}]e : T$.

Case [T-INVK]: $e = r.m(\bar{e}')$, $T = U$ Let $\Delta; \Gamma \vdash \bar{e}' : \bar{R}$. By application of the induction hypothesis, $\Delta; \emptyset \vdash [\bar{x} \mapsto \bar{d}]\bar{e}' : \bar{R}'$ where $\Delta \vdash \bar{R}' <: \bar{R}$. Also, let $\Delta; \Gamma \vdash r : V$. By the induction hypothesis, $\Delta; \emptyset \vdash [\bar{x} \mapsto \bar{d}]r : V'$ where $\Delta \vdash V' <: V$. But, because methods are not overloaded, and because of the type invariance restriction on [VALIDOVERRIDE], if $mtype(m, V) = \bar{T} \rightarrow U$, then $mtype(m, V') = \bar{T} \rightarrow U$, completing the case.

Lemma 4 (Fields are Preserved by Subtypes) *If $fields(U) = \bar{F}$ and $\emptyset \vdash V <: U$ then $\exists \bar{G}$ where $fields(V) = \bar{F} @ \bar{G}$.*

Proof We prove this by induction over the derivation for $fields(V)$.

Case $V = \text{Object}$: Then $U = \text{Object}$ and $\emptyset = \emptyset @ \emptyset$.

Case $V = \mathbf{C}(\bar{\mathbf{R}})$: Then $CT(\mathbf{C}) = \text{class } \mathbf{C}(\bar{\mathbf{X}} \triangleleft (\bar{\mathbf{I}}, \bar{\mathbf{J}})) : \mathbf{B} \triangleleft \mathbf{A}\{\dots\bar{\mathbf{H}}\dots\}$

Continue by induction on the derivation of $\emptyset \vdash V <: U$. The only interesting cases are [S-SUPER] and [S-TRANS].

Case [S-Super]: $[\bar{X} \mapsto \bar{R}]A = U$. Thus $\bar{G} = \bar{H}$.

Case [S-Trans]: Let T be the intermediate type. Then $fields(V) = fields(T) @ \bar{H}'$ and $fields(V) = fields(T) @ \bar{H}''$ by the induction hypothesis. Thus $\bar{G} = \bar{H}' @ \bar{H}''$.

Case $V = \text{typeOf}[\mathbf{C}(\bar{\mathbf{S}})]$: Then $CT(\mathbf{C}) = \text{class } \mathbf{C}(\bar{\mathbf{X}} \triangleleft (\bar{\mathbf{I}}, \bar{\mathbf{J}})) : \mathbf{B} \triangleleft \mathbf{A}\{\bar{w} \bar{h} \bar{e} \dots\}$. We continue by induction over the derivation of $\emptyset \vdash V <: U$. The only complex cases are [S-KIND] and [S-TRANS].

Case [S-Trans]: As above.

Case [S-Kind]: $[\bar{X} \mapsto \bar{S}]A = U$ Then $fields(V) = [\bar{X} \mapsto \bar{S}]\bar{w} \bar{h}$. By well-formedness of \mathbf{C} , we know that $fields(\mathbf{B}) \subseteq \bar{w} \bar{h}$. Then $[\bar{X} \mapsto \bar{S}]fields(\mathbf{B}) \subseteq [\bar{X} \mapsto \bar{S}]\bar{w} \bar{h}$, and $fields([\bar{X} \mapsto \bar{S}]\mathbf{B}) \subseteq [\bar{X} \mapsto \bar{S}]\bar{w} \bar{h}$ by lemma Substitution Distributes over *fields*.

Lemma 5 (Substitution Distributes over *fields*) $[\bar{X} \mapsto \bar{R}]fields(\mathbf{G}) = fields([\bar{X} \mapsto \bar{R}]\mathbf{G})$

Proof By induction over the derivation of $fields(\mathbf{G})$.

Case [F-Object]: Trivial.

Case [F-Class]: Let $\mathbf{G} = \mathbf{C}(\bar{\mathbf{S}})$. Then $CT(\mathbf{C}) = \mathbf{class} \ \mathbf{C}(\bar{\mathbf{Y}} \triangleleft (\bar{\mathbf{I}}, \bar{\mathbf{J}})) : \mathbf{B} \triangleleft \mathbf{A}\{\dots \bar{\mathbf{W}} \ \bar{\mathbf{g}}\}$, $fields(\bar{\mathbf{Y}} \mapsto \bar{\mathbf{S}}] \mathbf{A}) = \bar{\mathbf{U}} \ \bar{\mathbf{h}}$ and $fields(\mathbf{C}(\bar{\mathbf{S}})) = \bar{\mathbf{U}} \ \bar{\mathbf{h}} \ @ \ [\bar{\mathbf{Y}} \mapsto \bar{\mathbf{S}}] \bar{\mathbf{W}} \ \bar{\mathbf{g}}$.

Thus $[\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}]fields(\mathbf{C}(\bar{\mathbf{S}})) = [\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}](\bar{\mathbf{U}} \ \bar{\mathbf{h}} \ @ \ [\bar{\mathbf{Y}} \mapsto \bar{\mathbf{S}}] \bar{\mathbf{W}} \ \bar{\mathbf{g}}) = ([\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}] \bar{\mathbf{U}} \ \bar{\mathbf{h}}) \ @ \ ([\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}][\bar{\mathbf{Y}} \mapsto \bar{\mathbf{S}}] \bar{\mathbf{W}} \ \bar{\mathbf{g}}) = fields([\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}] \mathbf{C}(\bar{\mathbf{S}})).$

Case [F-Typeof]: In this case, $\mathbf{G} = \mathbf{typeof} \ [\mathbf{C}(\bar{\mathbf{S}})]$. Then $CT(\mathbf{C}) = \mathbf{class} \ \mathbf{C}(\bar{\mathbf{Y}} \triangleleft (\bar{\mathbf{I}}, \bar{\mathbf{J}})) : \mathbf{B} \triangleleft \mathbf{A}\{\bar{\mathbf{T}} \ \bar{\mathbf{f}} \ \bar{\mathbf{e}} \dots\}$ and $fields(\mathbf{typeof} \ [\mathbf{C}(\bar{\mathbf{S}})]) = [\bar{\mathbf{Y}} \mapsto \bar{\mathbf{S}}] \bar{\mathbf{T}} \ \bar{\mathbf{f}}$. Therefore, $[\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}]fields(\mathbf{typeof} \ [\mathbf{C}(\bar{\mathbf{S}})]) = [\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}][\bar{\mathbf{Y}} \mapsto \bar{\mathbf{S}}] \bar{\mathbf{T}} \ \bar{\mathbf{f}}$. But note that $fields([\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}] \mathbf{typeof} \ [\mathbf{C}(\bar{\mathbf{S}})]) = fields(\mathbf{typeof} \ [\mathbf{C}([\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}] \bar{\mathbf{S}})]) = [\bar{\mathbf{Y}} \mapsto [\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}] \bar{\mathbf{S}}] \bar{\mathbf{T}} \ \bar{\mathbf{f}} = [\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}][\bar{\mathbf{Y}} \mapsto \bar{\mathbf{S}}] \bar{\mathbf{T}} \ \bar{\mathbf{f}}$.

Lemma 6 (Subtyping Preserves Method Typing) *If $mtype(\mathbf{m}, \mathbf{U}) = \bar{\mathbf{T}} \rightarrow \mathbf{S}$ and $\emptyset \vdash \mathbf{V} <: \mathbf{U}$ then $mtype(\mathbf{m}, \mathbf{V}) = \bar{\mathbf{T}} \rightarrow \mathbf{S}$.*

Proof By induction over the derivation of $\emptyset \vdash \mathbf{V} <: \mathbf{U}$.

Case [S-Reflex]: Trivial.

Case [S-Bound]: Not applicable.

Case [S-Super]: In this case, $\mathbf{V} = \mathbf{C}(\bar{\mathbf{R}})$, where $CT(\mathbf{C}) = \mathbf{class} \ \mathbf{C}(\bar{\mathbf{X}} \triangleleft (\bar{\mathbf{I}}, \bar{\mathbf{J}})) : \mathbf{B} \triangleleft \mathbf{A}\{\dots \bar{\mathbf{M}}\}$ and $\mathbf{U} = [\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}] \mathbf{A}$. We need to show that $mtype(\mathbf{m}, \mathbf{C}(\bar{\mathbf{R}})) = \bar{\mathbf{T}} \rightarrow \mathbf{S}$. By lemma *methods* is well-defined, it suffices to show that $\mathbf{U} \ \mathbf{m} \ (\bar{\mathbf{T}} \ \bar{\mathbf{x}}) \ \{\mathbf{e}\} \in methods(\mathbf{C}(\bar{\mathbf{R}}))$.

We know that $\mathbf{U} \ \mathbf{m} \ (\bar{\mathbf{T}} \ \bar{\mathbf{x}}) \ \{\mathbf{e}\} \in methods(\mathbf{U})$. So, we proceed by induction on the derivation of $methods(\mathbf{U})$.

Case $\mathbf{U} = \mathbf{Object}$: Impossible, since \mathbf{Object} has no methods.

Case $\mathbf{U} = \mathbf{D}(\bar{\mathbf{W}})$: Then, $\mathbf{U} = [\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}] \mathbf{A}$. By [METHODSCONTEXT], $methods(\mathbf{V}) = methods(\mathbf{C}(\bar{\mathbf{R}})) = ([\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}] \bar{\mathbf{M}}) \cup methods([\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}] \mathbf{A}) = ([\bar{\mathbf{X}} \mapsto \bar{\mathbf{R}}] \bar{\mathbf{M}}) \cup methods(\mathbf{U})$. Therefore, any method in $methods(\mathbf{U})$ must be in $methods(\mathbf{V})$.

Case [S-Typeof]: Analogous to [S-SUPER].

Case [S-Trans]: Let the intermediate type be \mathbf{T} . Then $\emptyset \vdash \mathbf{V} <: \mathbf{T}$ and $\emptyset \vdash \mathbf{T} <: \mathbf{U}$. Thus, by the induction hypothesis, $mtype(\mathbf{m}, \mathbf{V}) = mtype(\mathbf{m}, \mathbf{T}) = mtype(\mathbf{m}, \mathbf{U})$.

Lemma 7 (mtype and mbody Agree) *If $mbody(\mathbf{m}, \mathbf{G}) = (\bar{\mathbf{x}}, \mathbf{e})$ and $mtype(\mathbf{m}, \mathbf{G}) = \bar{\mathbf{T}} \rightarrow \mathbf{U}$ then $\emptyset; \mathbf{this} : \mathbf{G}, \bar{\mathbf{X}} : \bar{\mathbf{T}} \vdash \mathbf{e} : \mathbf{U}'$ where $\emptyset \vdash \mathbf{U}' <: \mathbf{U}$.*

Proof Since the class table is well formed, then from the hypotheses we have that $mbody(\mathbf{m}, \mathbf{G}) = (\bar{\mathbf{x}}, \mathbf{e})$ and $mtype(\mathbf{m}, \mathbf{G}) = \bar{\mathbf{T}} \rightarrow \mathbf{U}$ which implies that $\mathbf{U} \ \mathbf{m} \ (\bar{\mathbf{T}} \ \bar{\mathbf{x}}) \ \{\mathbf{e}\} \in methods(\mathbf{G})$. Therefore, $\mathbf{U}' \ \mathbf{m} \ (\bar{\mathbf{T}}' \ \bar{\mathbf{x}}) \ \{\mathbf{e}'\}$ ok in $\mathbf{C}(\bar{\mathbf{X}})$ where $\mathbf{G} = \mathbf{C}(\bar{\mathbf{V}})$ for some $\bar{\mathbf{v}}$. Then we have the desired conclusion by lemma *Type Substitution Preserves Typing*.

Lemma 8 (methods is well-defined) *If $\mathbf{U} \ \mathbf{m} \ (\bar{\mathbf{T}} \ \bar{\mathbf{x}}) \ \{\mathbf{e}\} \in methods(\mathbf{G})$ and $\mathbf{U}' \ \mathbf{m} \ (\bar{\mathbf{T}}' \ \bar{\mathbf{x}}) \ \{\mathbf{e}\} \in methods(\mathbf{G})$ then $\mathbf{T} = \mathbf{T}'$ and $\mathbf{U} = \mathbf{U}'$.*

Proof Immediate from the side constraint on class tables that no two methods share the same name.

Lemma 9 (Type Substitution Preserves Type OKness) *If $\bar{\mathbf{X}} \triangleleft \bar{\mathbf{G}} \vdash \mathbf{T}$ ok and $\emptyset \vdash \bar{\mathbf{H}} <: \bar{\mathbf{G}}$ and $\emptyset \vdash \bar{\mathbf{H}}$ ok then $\emptyset \vdash [\bar{\mathbf{X}} \mapsto \bar{\mathbf{H}}] \mathbf{T}$ ok.*

Proof By induction over the derivation of $\bar{X} \triangleleft \bar{G} \vdash T$ ok.

Case [WF-Object]: Immediate.

Case [WF-TypeOf]: Immediate from the induction hypothesis.

Case [WF-Var]: In this case, $T = X_i$ and $[\bar{X} \mapsto \bar{H}]X_i = H_i$. And we are given that $\emptyset \vdash \bar{H}$ ok.

Case [WF-Class]: Immediate from lemma Type Substitution Preserves Subtyping and the induction hypothesis.

Lemma 10 (Agreement of fields and field-vals) *If $fields(G) = \bar{T} \bar{f}$ and $field-vals(G) = \bar{T}' \bar{f}' \bar{e}$ then $\bar{T} = \bar{T}'$, $\bar{f} = \bar{f}'$ and $\emptyset; \emptyset \vdash \bar{e} : \bar{S}$ where $\emptyset \vdash \bar{S} <: \bar{T}$.*

Proof The proof is exactly analagous to the proof of lemma *mtype* and *mbody* Agree.